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Notes on 'particle detectors'

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Abstract. This paper examines the theory of 'particle detectors' in quantum field theory on curved space–times. The historical background of the subject is discussed and then an example is given to show that an observer accelerating uniformly through the Minkowski vacuum can carry a detector which remains unexcited. The notion of a family of natural detectors for any given observer (based on an appropriate definition of rigidity) is introduced, and the response of such detectors is investigated. The theory is illustrated by consideration of a rotating detector in flat space–time. An extensive discussion is given of the radiation effects arising both from the walls of the detector and from the interaction with the external field. These effects have, in the past, led to some confusion in the interpretation of detector response; we clarify this interpretation here.

1. Introduction and historical background

In a generally covariant formulation of quantum field theory one can construct a Fock space corresponding to any complete orthonormal set of solutions to the wave equation. Thus, for each such set one can define states which correspond in conventional quantum field theory to particle states. Given so many definitions of a 'particle' it is of interest to ask in what way, if any, such states will correspond to anyone's natural concept of a particle. In the face of this question it is most appropriate to fall back on operational definitions and ask how a given 'particle detector' would respond in any given state. One can then try to relate this response to the description of that state in terms of the various 'particle' definitions.

The first such analysis of particle detectors in quantum field theory on a fixed background space–time was given by Unruh (1976). We note, however, that the corresponding problem had been considered earlier in the field of quantum optics (Glauber 1963). Unruh considered two simple model detectors, one non-relativistic and the other fully relativistic. Both detectors were spatially extended having components which followed trajectories of the Rindler Killing vector field with mean acceleration a. Unruh was able to show that these detectors would react on moving through Minkowski space–time in its natural vacuum state as if immersed in a thermal heat bath at local temperature $a/2\pi$.

Most subsequent analyses of particle detection, for example that of DeWitt (1979), have dealt with the response of pointlike detectors. However, in quantum field theory the concept of particle, as defined through Fock bases, is a global one. Therefore,

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the connection between the response of particle detectors and particles is somewhat obscure. Rather, one suspects that any full analysis of particle detection must require a discussion of detectors of finite spatial extent. There may, of course, exist regimes in which the finite detector behaves essentially as an idealised pointlike detector, but this can only be determined once one knows how a realistic detector responds.

Support for this view may be obtained from a consideration of the quantum field theory associated with conformal Killing vector fields in flat space–time (Brown *et al* 1982). First we consider an observer who is accelerating uniformly through flat space–time in the Minkowski vacuum state but who carries with him a finite detector different from that considered by Unruh. Specifically, we suppose that the walls of the detector follow trajectories of a field of the form of K_4, as defined by

$$K_4{}^a = (1+(t+x)^2+y^2+z^2, -1+(t+x)^2-y^2-z^2, 2(t+x)y, 2(t+x)z)$$

in some inertial frame and illustrated in figure 1. Further, let the external field be

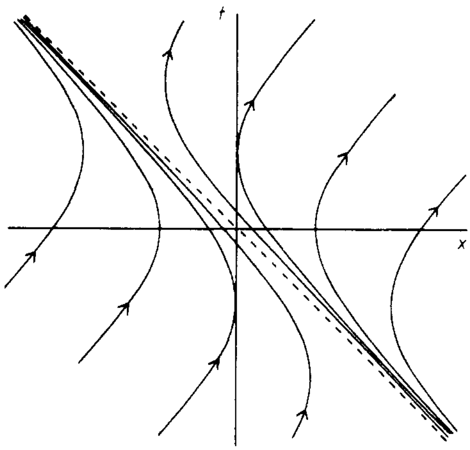

Figure 1. The trajectories of the flat space conformal Killing vector field 'K_4' in the plane $y=z=0$.

conformally invariant and let the detector field be conformally invariant and satisfy Dirichlet boundary conditions on the sides of the detector. Finally, let the coupling between the detector and the external field be conformally invariant. The response of this detector may now most simply be determined by making a conformal transformation of the form described in Brown *et al* (1982) which maps the space–time into another Minkowski space. The conformal image of the detector is just an inertial detector in the new space–time. Moreover, it was shown in Brown *et al* (1982) that under this conformal transformation the Minkowski vacuum is mapped into itself. Thus, the original detector is conformally equivalent to an inertial detector moving through the Minkowski vacuum. Hence, for example, if it starts in its ground state it must remain there. That is, an observer with constant acceleration can take with him a seemingly natural detector which remains unexcited on moving through the Minkowski vacuum.

A similar situation can be encountered, again in flat space, even when every worldline of two alternative detectors is inertial. Consider the Milne universe, which is the interior of the future light cone of some point P in Minkowski space. Let this space be in its natural vacuum state, the Milne vacuum. For an inertial observer who comes out of the 'initial singularity' at P there are two equally natural ways in which he can construct a detector. Either he can take his detector to be defined by neighbouring inertial trajectories that all emerge from P, or by parallel neighbouring inertial trajectories. Taking the fields and their couplings to be as in the previous example, we can perform a conformal transformation mapping the Milne universe into the open Einstein universe (Brown *et al* 1982). The first detector, whose conformal image is inertial, must remain in its ground state while the second detector may become excited.

In this paper we shall investigate the general problem of the construction of particle detectors of finite spatial extent. Unruh (1976) studied two types of spatially extended detector. However, both types had undesirable features—the first was non-relativistic and the second was unbounded. In § 2 we introduce an alternative fully relativistic model detector with finite spatial extent.

We consider measurements made on the state of matter in the universe with such a detector from the point of view of some classical point observer moving along a trajectory, γ. There are, of course, infinitely many ways in which the single trajectory, γ, can be extended to a finite detector. However, in § 2 we shall argue that not all such extensions are equally appropriate to discussions of particle detection. Further, we describe the construction of a preferred set of detectors for a general observer on an arbitrary space–time. Such detectors will have the property that they appear rigid to the given observer, although they will not in general appear so to other observers. This is the most natural generalisation of Unruh's prescription to arbitrary motions and we shall refer to such a detector as a particle detector corresponding to γ.

In the case when γ coincides with the trajectory of a Killing or conformal Killing vector field K it is also natural to consider a detector whose components follow neighbouring trajectories of K. The response of such detectors will be discussed in § 3.

The thermal response of Unruh's uniformly accelerating detector is in full accord with the description of the Minkowski vacuum as a state on Rindler space in terms of Rindler particles—it being a thermal state with local temperature $a/2\pi$. However, in other calculations performed subsequently, the corresponding result did not appear to hold. For example, a uniformly rotating detector moving through the Minkowski vacuum will be excited (Letaw 1981) even though it contains no 'rotating particles' (Denardo and Percacci 1978, Letaw and Pfautsch 1980). These results led Letaw and Pfautsch (1981) to conclude that:

'The correspondence between vacuum states defined via canonical quantum field theory and via a detector is thus broken for more general stationary motions, and we must conclude that the two definitions are inequivalent.'

In § 5 we shall argue that a detector whose components follow trajectories of a Killing vector field K does, in fact, respond to the presence of K particles. However, some care must be taken since the response will, in general, be complicated by the effects of radiation by the detector itself. These radiation effects will depend on the precise nature of the construction of the detector and on its coupling to the external field. As with any 'complete' measurement, when interpreting the result of a series of 'elementary' measurements one should exclude spurious, though sometimes interesting, effects due to the presence and nature of the measuring apparatus itself. If this is done here then the apparent discrepancy between particle detector measurements and canonical quantisation is removed.

2. General theory

For the reasons laid out in the introduction, we now wish to discuss the theory of particle detectors of finite spatial extent. In this section we will consider the theory on a general space–time $\{M, g\}$ before specialising in § 3 to spaces possessing a conformal Killing vector field. The size of the detector is arbitrary except that we assume, of course, that all points of it follow timelike trajectories.

We suppose that there is a real scalar field Φ on $\{M, g\}$, which we will refer to as the external field. We take the detector to be a resonant cavity containing another real scalar field ψ which satisfies Robin boundary conditions on the walls of the cavity. Finally, we suppose that there is a weak coupling between the two fields described by the interaction Lagrangian $L_{\mathrm{int}} = \lambda\psi\Phi$.

Suppose that initially the detector field is in the state $|A\rangle_{\mathrm{D}}$, and the external field is in the state $|G\rangle$. Transitions will be induced in the detector by interaction with the external field. We shall calculate the probability that the detector undergoes a transition to the state $|B\rangle_{\mathrm{D}}$, which for convenience we shall take to be orthogonal to $|A\rangle_{\mathrm{D}}$. The amplitude for such a transition in the detector together with a corresponding change in the state of the external field to the state $|F\rangle$, say, is given by perturbation theory to be

$$\mathscr{A}(F, B) = \mathrm{i}\lambda \int \mathrm{d}^4x\, \sqrt{g(x)}\,{}_{\mathrm{D}}\langle B|\hat{\psi}(x)|A\rangle_{\mathrm{D}}\langle F|\hat{\Phi}(x)|G\rangle + \mathrm{O}(\lambda^2). \tag{2.1}$$

The integration here is taken over the region of the worldtube of the detector in which the observation is made. This region can have finite temporal extent if the coupling is only switched on for some specified period. To avoid spurious excitations this switching should be performed adiabatically.

We stress that our interest above is to observe excitations in the detector due to the interaction of the detector field with the external field; we do this by working in the interaction picture. In certain cases (DeWitt 1975) one can define asymptotic $|\mathrm{in}\rangle_{\mathrm{D}}$ and $|\mathrm{out}\rangle_{\mathrm{D}}$ vacuum states in the detector, and the in-vacuum may contain out-particles in which case one says that particle production has occurred. Even if there is no coupling between the detector and the external field the description of the state of the detector may change in this way but, of course, the state itself will not alter. Comparison with this case will enable one to distinguish between the effects of actual changes in state brought about by the interaction and mere changes of description of a fixed state. Clearly it is the former which is relevant to any discussion of the detection of particles of the external field.

The total probability that the detector ends up in the state $|B\rangle_{\mathrm{D}}$ is then given to lowest order in λ by

$$\begin{aligned} P_{\mathrm{B}} &= \sum_{|F\rangle} |\mathscr{A}(F, B)|^2 \\ &= \lambda^2 \int\!\!\int \mathrm{d}^4x\, \mathrm{d}^4x'\, \sqrt{g(x)}\sqrt{g(x')}\,{}_{\mathrm{D}}\langle A|\hat{\psi}(x)|B\rangle_{\mathrm{D}} \\ &\qquad \times {}_{\mathrm{D}}\langle B|\hat{\psi}(x')|A\rangle_{\mathrm{D}}\langle G|\hat{\Phi}(x)\hat{\Phi}(x')|G\rangle \end{aligned} \tag{2.2}$$

where the sum is over a complete set of states for the free Φ field.

$\langle G|\hat{\Phi}(x)\hat{\Phi}(x')|G\rangle$ is the two-point Wightman function for the external field in the state $|G\rangle$. Thus, for example, if $\{M, g\}$ is some flat space–time and $|G\rangle$ is the Minkowski vacuum state, $|M\rangle$, then we can use the known analytic form for the Minkowski–Wightman function, given in inertial coordinates $(T, \boldsymbol{X})$ by

$$\langle M|\hat{\Phi}(x)\hat{\Phi}(x')|M\rangle = \{2\pi^2[-(T-T'-\mathrm{i}\varepsilon)^2 + (\boldsymbol{X}-\boldsymbol{X}')^2]\}^{-1}. \tag{2.3}$$

Equation (2.2) yields the probability of a transition between any two orthogonal states in an arbitrarily shaped detector on any space–time. Now we must address the problem of the meaning that is to be attached to this result in terms of particle detection. This problem is related to the specification of a preferred class of detectors for an observer on a particular trajectory γ. To discuss this, we suppose for the moment that the detector is not coupled to the external field.

First, we consider an inertial observer in Minkowski space for which we have a well defined notion of particle. If such an observer is placed between two plane mirrors which follow arbitrary trajectories in space–time then he will, in general, see the mirrors radiating both a flux of particles and a flux of energy (Fulling and Davies 1976, Ford and Vilenkin 1982). Only in the special case of two inertial mirrors will there be neither a flux of particles nor a flux of energy. Now suppose that we consider these mirrors as being the walls of a model particle detector. It is then clear that if an inertial observer wishes to determine the particle content of some state it is most convenient for him to use a detector with walls that are fixed relative to himself. If the detector is not rigid in this sense, then when interpreting his readings the observer must allow for the effects of the radiation emitted by the walls of the detector itself. Clearly, these effects, which depend on details of the structure of the detector, are irrelevant to the central issue of particle detection and should be avoided whenever possible.

Similar comments apply to an observer who follows a trajectory of the Killing vector field, K, of a static universe, for which there again exists a natural notion of particle. This observer will naturally perform measurements with a detector whose walls do not vary with time. It is only for such detectors that the walls do not, in his opinion, emit particles.

For an arbitrary space–time and a general observer one does not, *a priori*, have any well defined notion of particle. However, we shall now argue there always exists a preferred class of detectors for such an observer which incorporates the important features of the detectors of the above examples. The most obvious property that we would like a member of this preferred class of detectors to possess is that it should be rigid in some appropriate sense. The standard definition of rigidity is that given by Born (1909). If V denotes the velocity field of a body then that body is rigid if its expansion tensor vanishes, that is, if

$$\theta_{ab} \equiv h_{ac}h_{bd}V^{(c;d)} = 0 \tag{2.4}$$

where $h_{ab} = g_{ab} + V_aV_b$. Unfortunately, as is well known (Trautman 1965), even in flat space–time it is impossible given an arbitrary timelike trajectory to find a rigid body such that one of its particles follows the given trajectory.

However, the above condition is rather more stringent than we require since we are only really interested in how the observer on the given trajectory γ views his detector. The natural coordinates with which to describe the system are, therefore, those defined by a proper reference frame associated with γ (Misner *et al* 1973): At each point of γ send out geodesics orthogonal to γ and assign coordinates to events in a neighbourhood of γ by using the proper time τ at which the geodesic was sent out, the proper distance s along the geodesic and the spherical polar angles of its tangent vector at γ as measured in a suitably transported orthonormal tetrad $\{e_\alpha\}$. The transport law for the tetrad is given by

$$(V^a\nabla_a)e_\alpha = -\Omega\cdot e_\alpha, \qquad \Omega_{ab} = A_aV_b - V_aA_b + \eta_{abcd}V^cW^d,$$

where V is the velocity of the observer, A his acceleration and W the angular velocity of the spatial basis vectors $\{e_1, e_2, e_3\}$ relative to Fermi–Walker transported vectors. W is arbitrary, but merely describes a rigid rotation of the *spatial* basis vectors, which simply corresponds to a change of coordinates in the three-surfaces τ = constant.

Clearly, in general, these coordinates will only be well defined in a limited neighbourhood of γ since the spacelike geodesics will converge away from γ. The surfaces τ = constant in this construction represent the surfaces of simultaneity for the observer and are therefore the natural surfaces that the observer would use in constructing his quantum field theory (Misner *et al* 1973). An observer travelling on γ will naturally say that a body is rigid if there exists a choice of W for which the corresponding spatial coordinates of the components of the body frame remain fixed. We will describe such a body as γ-rigid. In the special case when γ is a Killing trajectory, a body which follows other trajectories of the Killing vector field will be both Born rigid and γ-rigid.

To an observer moving along γ, the walls of a γ-rigid detector appear fixed so that they should not, in his opinion, be emitting particles (although from any other observer's viewpoint they may be). Therefore, this observer on γ will naturally discuss 'particle detection' in terms of such detectors. In this way the observer's concept of particle detection (and hence of particle) is determined by the response of this preferred class of detectors. The distinguishing feature of such detectors is that when they are uncoupled there are no spurious radiation effects; in § 5 we shall see that when the detector is coupled to the external field additional spurious radiation effects arise.

The problem of the quantisation of the field in a γ-rigid detector can be addressed by regarding the detector as a space–time in its own right. Unfortunately, this means that for the case of a general motion the problem is just as hard as that of quantising the field in a general space–time. Thus, to make any further progress we must make additional simplifying assumptions. We shall do this in § 3; however, in passing, we note here that as our construction of γ-rigid detectors defines a preferred form $d\tau$ on the detector space–time one might think of adopting the 'generalised Wick rotation' scheme of Candelas and Raine (1977). Unfortunately, in general, the physical significance of this procedure is obscure and so we shall not discuss it further.

3. Stationary detectors

To gain some insight into the rather complicated expressions of § 2, we now suppose that space–time possesses a stationary region, with a timelike Killing vector field K. If one wishes to consider the observations of an observer who follows a trajectory of K then it is natural to choose a finite detector which is related to K. We will describe such a detector as stationary. As noted above, such a detector is both Born rigid and γ-rigid.

Let t be the time coordinate associated with K, in the sense that $K^a\partial_a = \partial_t$. The detector will now be taken to satisfy any set of time-independent boundary conditions. In such a detector we can choose a complete orthonormal set of modes of the form $\exp(-i\varepsilon t)f_\alpha(\boldsymbol{x})$ (DeWitt 1975). Let $\{u_\alpha(t, \boldsymbol{x})\}$ be such a set and perform the quantisation in the standard fashion. We denote the corresponding vacuum state by $|0\rangle_D$ and the annihilation operator corresponding to the mode $u_\alpha(t, \boldsymbol{x}) = \exp(-\mathrm{i}\varepsilon t) f_\alpha(\boldsymbol{x})$ by $\hat{a}_\alpha$.

In stationary regions of space–time, special interest is attached to states which respect the stationarity in the sense that they are invariant under the time translation operator generated by t. Let us suppose that $|G\rangle$ is such a state; then we have

$$\langle G|\hat{\Phi}(t, \boldsymbol{x})\hat{\Phi}(t, \boldsymbol{x}')|G\rangle = \langle G|\hat{\Phi}(t-t', \boldsymbol{x})\hat{\Phi}(0, \boldsymbol{x}')|G\rangle.$$

Using equation (2.2) we can now calculate the probability for the detector to undergo a transition from its vacuum state, $|0\rangle_D$, to a 'one-particle state' $|\alpha\rangle = \hat{a}^\dagger_\alpha|0\rangle_D$. It is convenient to introduce new variables $s = t - t'$ and $s' = \frac{1}{2}(t+t')$. The integrand in (2.2) does not depend on s' ($\sqrt{g(t, \boldsymbol{x})} = \sqrt{g(\boldsymbol{x})}$ by stationarity). The s' integration is thus a trivial integration over all time which arises since (as a consequence of the stationarity) there is a constant probability per unit time of a transition. This constant transition probability rate, $\dot{P}_\alpha$, can readily be identified as

$$\dot{P}_\alpha = \lambda^2 \int_{-\infty}^{\infty} \mathrm{d}s \exp(-\mathrm{i}\varepsilon s) \int\int \mathrm{d}^3\boldsymbol{x}\, \mathrm{d}^3\boldsymbol{x}' \sqrt{g(\boldsymbol{x})}\sqrt{g(\boldsymbol{x}')} f^*_\alpha(\boldsymbol{x}) f_\alpha(\boldsymbol{x}') \times \langle G|\hat{\Phi}(s, \boldsymbol{x})\hat{\Phi}(0, \boldsymbol{x}')|G\rangle. \tag{3.1}$$

It is often convenient to consider, rather than the transition probability rate $\dot{P}_\alpha$ to a particular state $|\alpha\rangle_D$, the transition rate, $\dot{P}_\varepsilon\, \mathrm{d}\varepsilon$, to a detector state with energy lying in the range ε to $\varepsilon + \mathrm{d}\varepsilon$. This can be obtained immediately from equation (3.1): if we let $\rho(\alpha|\varepsilon)$ denote the density of detector states with energy ε and generalised mode index α then

$$\dot{P}_\varepsilon = \lambda^2 \int_{-\infty}^{\infty} \mathrm{d}s \exp(-\mathrm{i}\varepsilon s) \int\int \mathrm{d}^3\boldsymbol{x}\, \mathrm{d}^3\boldsymbol{x}' \sqrt{g(\boldsymbol{x})}\sqrt{g(\boldsymbol{x}')} \Big(\sum_\alpha \rho(\alpha|\varepsilon) f^*_\alpha(\boldsymbol{x}) f_\alpha(\boldsymbol{x}') \Big) \times \langle G|\hat{\Phi}(s, \boldsymbol{x})\hat{\Phi}(0, \boldsymbol{x}')|G\rangle. \tag{3.2}$$

Suppose now that the space–time possesses only a conformal Killing vector field K, but that the external field is conformally invariant. Following the discussion of § 1, we can consider the response of a detector whose components follow trajectories of K. This detector will not be rigid even in the less restrictive trajectory dependent sense of § 2. However, it will be a natural detector for observers following trajectories of K to use; for example, in a Robertson–Walker universe this will be precisely the detector constructed by a set of comoving observers. Such detectors have also been studied by Sanchez (1981) for accelerated motions in two-dimensional Minkowski space–time,although no mention is made in this work of the peculiar non-rigidity of such detectors or their limited applicability in four dimensions.

If we choose the detector and coupling to be conformally invariant then, following the approach described in § 1, we may make a conformal transformation to a related stationary space–time. The conformal image of this detector will be a detector following trajectories of the Killing vector field. In § 5 we shall argue that such a detector will respond to the presence of particles of the natural (Killing vector based) quantisation. Here, correspondingly, the original detector will respond to the presence of particles of the natural (conformal Killing vector based) quantisation.

Clearly, as it stands, equation (3.1) is rather complicated and it is appropriate to ask whether there is any regime in which it can be approximated. A natural and commonly used approximation scheme is the monopole approximation in which the term $\langle G|\hat{\Phi}(t-t', \boldsymbol{x})\hat{\Phi}(0, \boldsymbol{x}')|G\rangle$ is replaced by $\langle G|\hat{\Phi}(t-t', \bar{\boldsymbol{x}})\hat{\Phi}(0, \bar{\boldsymbol{x}})|G\rangle$, where $\bar{\boldsymbol{x}}$ is the spatial coordinate of γ. Loosely speaking, the assumption being made is that the modes of the external field which can interact with the detector mode α are slowly varying over the dimensions of the detector. In other words, we are dealing with a long-wavelength or low-energy approximation.

With this approximation equation (3.1) becomes

$$\dot{P}_\alpha = \lambda^2 |\sigma_\alpha|^2 \int_{-\infty}^{\infty} ds \exp(-i\varepsilon s)\langle G|\hat{\Phi}(s)\hat{\Phi}(0)|G\rangle \tag{3.3}$$

where $\hat{\Phi}(t)$ denotes the value of $\hat{\Phi}$ on γ at time t, and

$$\sigma_\alpha \equiv \int d^3\boldsymbol{x}\, \sqrt{g(\boldsymbol{x})} f_\alpha(\boldsymbol{x}).$$

To find P_ε we need merely replace $|\sigma_\alpha|^2$ by $|\sigma_\varepsilon|^2 \equiv \Sigma_\alpha\, \rho(\alpha|E)|\sigma_\alpha|^2$. Equation (3.3) is precisely the transition probability for a point monopole $\hat{m}(t)$ coupled to the external field by the interaction $\lambda m(t)\Phi(t)$ (DeWitt 1979) with the identification

$$_{\mathrm{D}}\langle 0|\hat{m}(0)|\alpha\rangle_{\mathrm{D}} = \int d^3\boldsymbol{x}\, \sqrt{g(\boldsymbol{x})} f_\alpha(\boldsymbol{x}).$$

We note that in this approximation the cross-section factor, $|\sigma_\varepsilon|^2$, is independent of the external field mode with which the detector interacts.

The problem of when a monopole of the type studied by DeWitt (1979) represents a good approximation to any form of realistic detector on a general space–time is less clear. However, the assumption that one can separate out the time dependence of the detector modes, as DeWitt does, requires that space–time can be taken to be stationary over the duration of the observation.

4. The rotating detector

To illustrate the use of the general formulae and ideas of § 3 we now discuss the special case of a rotating detector in flat space–time. This example will also prove extremely useful for our discussions of the interpretation of detector response in § 5. By a 'rotating detector' we mean a rigid detector corresponding to an observer in Minkowski space who revolves about a fixed axis at constant radius a with constant angular velocity Ω, where, of course, $\Omega a < 1$. Such an observer lies on an integral curve of the Killing vector field $K^a \partial_a = \partial_T + \Omega \partial_\Theta$, where the cylindrical polar coordinates (T, R, Θ, Z) define some inertial frame. It follows that 'rotating detectors' are also rigid in the Born sense.

The discussion of a rotating system is simplified by the introduction of rotating polar coordinates. Denoting, as above, the cylindrical polar coordinates in some inertial frame by (T, R, Θ, Z), the related rotating coordinates are defined by the equations

$$t = T, \qquad r = R, \qquad \theta = \Theta - \Omega T, \qquad z = Z.$$

In these coordinates $K^a \partial_a = \partial_t$, the Minkowski line element is

$$ds^2 = -(1 - \Omega^2 r^2)\, dt^2 + 2\Omega r^2\, d\theta\, dt + r^2\, d\theta^2 + dr^2 + dz^2$$

and the scalar wave equation is

$$\left[-\left(\frac{\partial}{\partial t}-\Omega\frac{\partial}{\partial \theta}\right)^2+\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2}\right]\Phi=0. \tag{4.1}$$

We note that it is possible to perform a global quantisation on Minkowski space using modes adapted to K: a complete orthonormal set of modes satisfying equation (4.1) is given by

$$\Phi_{KMQ}=(1/2\pi(2W)^{1/2})\exp[-\mathrm{i}(W-M\Omega)t]\exp(\mathrm{i}M\theta)\exp(\mathrm{i}Kz)J_{|M|}(Qr) \tag{4.2}$$

where M is an integer, $Q\geqslant 0$, $W=(K^2+Q^2)^{1/2}$ and J_n is the Bessel function of the first kind of order n. The energy, E, of this mode, as measured by the rotating observer and defined through the equation $\mathscr{L}_K\Phi_{KMQ}=-\mathrm{i}E\Phi_{KMQ}$, is given by $E=W-M\Omega$. W is the energy of the mode as measured by an inertial observer; we shall, following convention, refer to it as the frequency of the mode Φ_{KMQ}. It is important to note that, although W must be positive, E can take either sign. (This idea is familiar from, for example, discussions of superradiance by Kerr black holes.)

The field can be quantised by imposing the standard canonical commutation relations (Denardo and Percacci 1978, Letaw and Pfautsch 1980), and the resulting vacuum state is found to be just the Minkowski vacuum. Indeed, the modes (4.2) are just standard cylindrical Minkowski modes written in rotating coordinates, so that even a rotating observer's natural definition of a particle coincides with that of an inertial observer who uses fixed cylindrical polar coordinates.

We consider a detector whose walls consist of the planes $\theta=\pm\alpha$, $z=\pm h$ and the cylinders $r=a$ and $r=b$, as illustrated in figure 2. It is very hard to construct modes

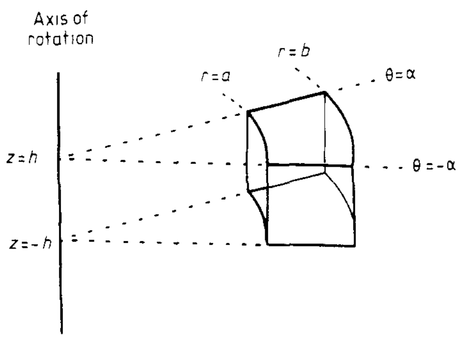

Figure 2. The rotating detector is taken to be bounded by the planes $\theta=\pm\alpha$, $z=\pm h$ and the cylinders $r=a$, $r=b$.

which vanish on all walls of the detector (Pfautsch 1981). However, since the configuration is stationary we can choose more general stationary boundary conditions: in particular, we impose periodic boundary conditions on the sides of the wedge and retain vanishing boundary conditions on the other faces. The detector modes can then be written

$$\psi_{kmq}=N_{kmq}\exp[-\mathrm{i}(\omega-m\Omega)t]\exp(\mathrm{i}m\theta)\sin(k[z+h])C_{|m|}(qr) \tag{4.3}$$

where $m=\pi j/\alpha$, $j\in\mathbb{Z}$; $k=\pi n/2h$, $n=0,1,2,\ldots$; $\omega=(q^2+k^2)^{1/2}$ and N_{kmq} is a normalisation factor. $C_{|m|}(qr)$ is a cylinder function of order $|m|$ (that is, $C_{|m|}(qr)=$

$\cos\beta\, J_{|m|}(qr)+\sin\beta\, Y_{|m|}(qr)$, for some β) such that $C_{|m|}(qa)=C_{|m|}(qb)=0$. We denote by ε the energy of the mode as measured by the rotating observer, $\varepsilon=\omega-m\Omega$. It is a remarkable fact, which we prove in the appendix, that the energy, ε, of each of the positive norm modes ψ_{kmq} in the detector is positive.

To apply the general formula (3.2) for the transition probability rate, $\dot{P}_\varepsilon$, we further require the density of states $\rho(jkq|\varepsilon)$. In the appendix we describe how to obtain an approximate form for this density of states:

$$\rho(jkq|\varepsilon)\propto\omega\theta(\omega)\theta(a\omega-\pi|j|/\alpha)\cos^{-1}(\pi|j|/\alpha\omega a). \tag{4.4}$$

We note that for given ε there are finite maximum and minimum values which j can take; we denote them by $j_{\max}$ and $j_{\min}$ respectively. They are determined by the inequality

$$-\frac{\alpha}{\pi}\frac{\varepsilon a}{1+\Omega a}\leqslant j\leqslant\frac{\alpha}{\pi}\frac{\varepsilon a}{1-\Omega a}.$$

This is illustrated in figure 3.

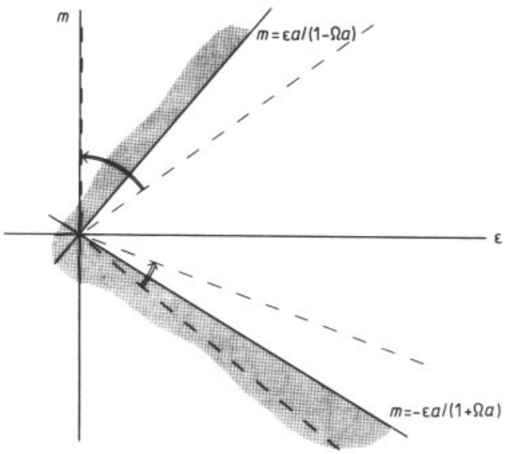

Figure 3. The allowed values of m and ε for the rotating detector. The disallowed area is shaded and the dependence on the angular velocity, Ω, is indicated by arrows from $\Omega a=0$ to $\Omega a=1$.

We can now use equation (4.4) together with the modes (4.2) and (4.3) in equation (3.2) to obtain an expression for the transition probability rate, $\dot{P}_\varepsilon$. The expression obtained is extremely long, and rather than present it in this form we choose to simplify it first by performing the standard monopole approximation in the r and z directions (which are uninteresting compared with the θ direction). In order that this approximation be valid we must assume that the radial and vertical dimensions of the detector are small in the sense discussed in § 3. However, we make no assumptions about the angular extent of the detector. The resulting expression for the transition probability rate per unit volume is:

$$\dot{P}_\varepsilon\propto\lambda^2\sum_{j_{\min}}^{j_{\max}}\left(\varepsilon-\frac{\pi j\Omega}{\alpha}\right)\cos^{-1}\left(\frac{\pi j}{\varepsilon a\alpha-\pi j\Omega a}\right)\mathscr{P}_j(\varepsilon,\alpha) \tag{4.5}$$

with

$$\mathscr{P}_j(\varepsilon, \alpha) = \sum_M (M\Omega - \varepsilon) \frac{\sin^2(\pi j + M\alpha)}{(\pi j + M\alpha)^2} \int_0^{\pi/2} d\theta \cos\theta J_M^2((M\Omega - \varepsilon)a \cos\theta) \tag{4.6}$$

the sum being from $M = [\varepsilon/\Omega] + 1$ to ∞, where $[x]$ denotes the integer part of x.

In the limit as the angular extent of the detector tends to zero only terms with $j = 0$ survive in the sum (4.5). This behaviour is an artefact of the use of periodic boundary conditions. The limiting response for a detector of this type is then given by

$$\dot{P}_\varepsilon \propto \lambda^2 \varepsilon \sum_{[\varepsilon/\Omega]+1}^{\infty} (M\Omega - \varepsilon) \int_0^{\pi/2} d\theta \cos\theta J_M^2((M\Omega - \varepsilon)a \cos\theta)$$

$$\propto \lambda^2 \varepsilon \int_{-\infty}^{\infty} ds \exp(-i\varepsilon s) \left[-\left(\frac{s - i\epsilon}{\gamma}\right)^2 + 4a^2 \sin^2\left(\frac{\Omega(s - i\epsilon)}{2\gamma}\right)\right]^{-1} \tag{4.7}$$

where $\gamma = (1 - \Omega^2 a^2)^{-1/2}$. The second equality follows from (2.3) on expressing the Minkowski Wightman function in terms of the modes (4.2). The results of a numerical integration of (4.7) are displayed in figure 4 for the case $\Omega a = 0.5$.

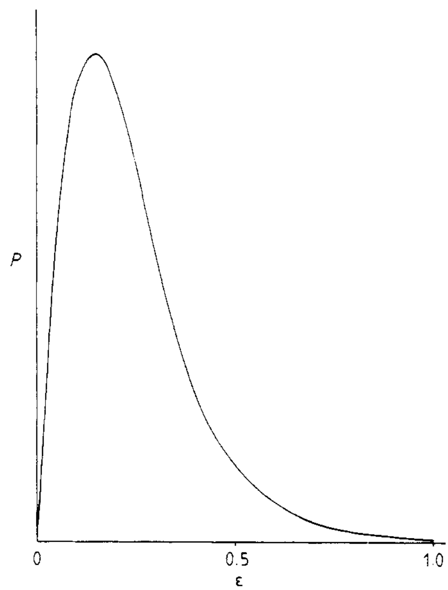

Figure 4. The transition probability rate for an infinitesimal rotating detector with $\Omega a = 0.5$.

Equation (4.7) shows that there is a non-zero probability of a rotating detector becoming excited even on moving through its associated vacuum state. Moreover, it does so in a non-thermal way despite feeling a constant acceleration. The interpretation of this result, which led to considerable confusion in the literature, will be discussed in § 5.

We note that the integrand in (4.7) possesses a pole infinitesimally displaced from the origin. This pole produces a term in $\dot{P}_\varepsilon$ proportional to $\theta(-\varepsilon)$. Such a term is present even when the detector moves inertially and ensures that the energy in the detector does not increase without limit.

Equation (4.5) can also be used to determine the response of a detector of non-zero angular extent. The results of a numerical computation of $\dot{P}_\varepsilon$ for a detector with $\alpha = \frac{11}{7}$ and $\Omega a = 0.8$ are displayed in figure 5. This figure clearly displays how, when α is finite, states with non-zero angular momentum can be excited in the detector and contribute to the total probability. These transitions are suppressed in the monopole limit; for example, states with $j = 1$ only contribute above an energy $\varepsilon = (1 - \Omega a)\pi/(a\alpha)$ and this clearly tends to infinity as $\alpha \to 0$. We note that states with $j = -1$ only contribute above the higher energy $\varepsilon = (1 + \Omega a)\pi/(a\alpha)$. We shall return to this point in § 5.

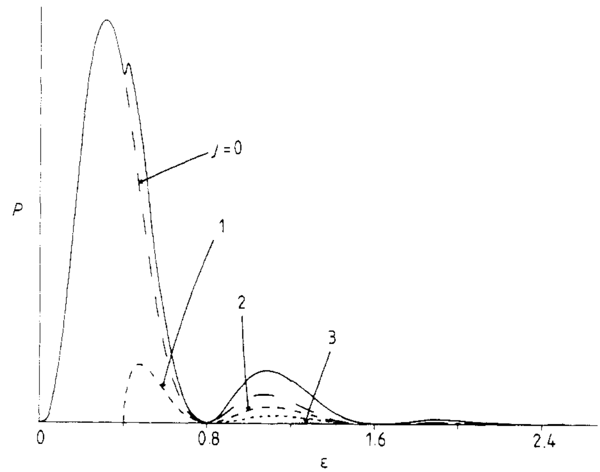

Figure 5. The transition probability rate for a finite rotating detector with $\alpha = \frac{11}{7}$ and $\Omega a = 0.8$. The full curve indicates the total transition probability rate while the various broken and dotted curves indicate the transition probability rate to a state of particular angular quantum number, j.

5. Interpretation

In § 3 we showed that in the monopole approximation the response of a stationary detector with simple linear coupling is essentially determined by the Fourier transform of the autocorrelation function of the external field—the transform being taken with respect to proper time along the worldline of the detector. This result admits an interesting interpretation (Sciama *et al* 1981): the Fourier transform of the autocorrelation function is directly related, by the Wiener–Khinchin theorem, to the power spectrum of the fluctuations of the field. Thus, the detector can be said to be acting as a 'fluctuometer'.

In § 4 we showed that for a rotating detector the connection between fluctuometer response and particle detection cannot be direct: the rotating detector became excited even though no particles were present. Indeed, the view has often been expressed that the detector is *just* acting as a vacuum fluctuometer and that one should not expect any information from it on the particle content of space–time. However, in this section we wish to discuss how the response of a simple detector can be interpreted to yield information about the particle content of a state. The important distinction we wish to make here is between the 'elementary' and 'complete' levels of measurement

(DeWitt 1965). The elementary level, at which our discussions have been based so far, deals merely with the simple relationship between the system on which the measurement is being performed and the response of the measuring apparatus. On the other hand, the complete level is concerned with the interpretation and physical conclusions that can be drawn from this response, and as such usually deals with a sequence of elementary measurements. In our case, at an elementary level, we have shown that a point monopole will act as a fluctuometer. However, more realistic finite detectors or even point monopoles with a different coupling will not respond in this way. These differences at the elementary level are unimportant and to be expected. Rather, it is the conclusions drawn at the complete level, which in our case concern the particle content of space-time, that we expect to be of fundamental, model-independent significance.

For simplicity we suppose that there exists a global Killing vector field K on the space-time which is timelike somewhere. We wish to determine how the response of a stationary detector is related to the particle content of the space-time. We stress that in many cases of interest K is not timelike over all of space. For example, the Rindler Killing vector field is timelike for $|x|>|t|$ but is spacelike for $|x|<|t|$, while the rotating Killing vector field is timelike for $\Omega r<1$ but spacelike for $\Omega r>1$.

We start by introducing coordinates related to K in the usual way. These coordinates may possess (unimportant) singularities on the horizons of K, but we assume that they are otherwise well defined over the whole of space-time. Such coordinates can certainly be chosen for all flat space Killing vector fields (Letaw and Pfautsch 1981). It is then possible to introduce a complete orthonormal set of modes for the space-time which have the form $\chi_j=\exp(-\mathrm{i}E_jt)p_j(\boldsymbol{x})$. In stationary regions of the space-time E_j may be identified with the energy of the mode. In general the requirement of positive norm is not equivalent to the condition that E_j be positive. However, it will be the same if space-time is static and the surfaces $t=$ constant are Cauchy surfaces; this is the case for the Rindler Killing vector field.

We can use the above set of modes to construct a Fock space for the quantum theory of the external field. It is important to note that in the formalism of canonical quantisation one must associate a creation operator with a positive *norm* solution to the wave equation. Decomposing the field operator as $\hat{\Phi}=\hat{b}_j\chi_j+\hat{b}_j^\dagger\chi_j^*$ we define the vacuum state that is naturally associated with K by the equations

$$\hat{b}_j|K\rangle=0 \qquad \forall j, \qquad \langle K|K\rangle=1.$$

Let us now assume, as in § 3, that the state, $|G\rangle$, of the external field is invariant under the action of K. We stress, however, that we are *not* assuming that $|G\rangle$ is the state constructed from the modes χ_j. In fact, let us suppose that $|G\rangle$ is the vacuum state of a Fock space constructed from the positive norm modes $\{\Phi_m\}$. Decomposing the field operator as $\hat{\Phi}=\hat{a}_m\Phi_m+\hat{a}_m^\dagger\Phi_m^*$, the state $|G\rangle$ is thus defined by the equations

$$\hat{a}_m|G\rangle=0 \qquad \forall m, \qquad \langle G|G\rangle=1.$$

For example, in Minkowski space we could have $|G\rangle=|M\rangle$ and take Φ_m to be plane wave modes while χ_j might be Rindler modes or rotating modes.

Finally, we note that, as $\{\chi_j\}$ and $\{\Phi_m\}$ are both complete sets, there exists a Bogoliubov transformation between them. Let this be given by the equation

$$\Phi_m=\alpha_{mj}\chi_j+\beta_{mj}\chi_j^*. \tag{5.1}$$

Now, as above, let us consider a transition in which the detector moves from its vacuum state to the one-particle state associated with the mode $\exp(-\mathrm{i}\varepsilon t)f_\alpha(\boldsymbol{x})$, while the external field goes from the state $|G\rangle$ to the state $|F\rangle$. In terms of the notation of this section the amplitude (2.2) for the detector to undergo such a transition is

$$\mathscr{A}(F,\alpha)=\mathrm{i}\lambda\int_{-\infty}^{\infty}\mathrm{d}s\,\exp(-\mathrm{i}\varepsilon s)\int\mathrm{d}^3\boldsymbol{x}\,\sqrt{g(\boldsymbol{x})}f_\alpha^*(\boldsymbol{x})\langle F|\hat{\Phi}(s,\boldsymbol{x})|G\rangle \tag{5.2}$$

and so, assuming that the time and frequency integrals can be interchanged (a procedure which is certainly valid in the Rindler and rotating cases),

$$\mathscr{A}(F,\alpha)=\mathrm{i}\lambda[\delta(\varepsilon+E_j)\langle F|\alpha_{mj}^*\hat{a}_m^\dagger|G\rangle A_{j\alpha}^*+\delta(\varepsilon-E_j)\langle F|\beta_{mj}^*\hat{a}_m^\dagger|G\rangle B_{j\alpha}^*] \tag{5.3}$$

$$=\mathrm{i}\lambda[\delta(\varepsilon+E_j)\langle F|\hat{b}_j^\dagger|G\rangle A_{j\alpha}^*+\delta(\varepsilon-E_j)\langle F|\hat{b}_j|G\rangle B_{j\alpha}^*] \tag{5.4}$$

where

$$A_{j\alpha}=2\pi\int\mathrm{d}^3\boldsymbol{x}\,\sqrt{g(\boldsymbol{x})}p_j(\boldsymbol{x})f_\alpha(\boldsymbol{x}),\qquad B_{j\alpha}=2\pi\int\mathrm{d}^3\boldsymbol{x}\,\sqrt{g(\boldsymbol{x})}p_j^*(\boldsymbol{x})f_\alpha(\boldsymbol{x}). \tag{5.5}$$

The total probability of a transition to a detector state can now be written as

$$P_\alpha=\lambda^2[\delta(\varepsilon+E_j)\delta(\varepsilon+E_k)\alpha_{mj}\alpha_{mk}^*A_{j\alpha}A_{k\alpha}^*+2\delta(\varepsilon+E_j)\delta(\varepsilon-E_k)\,\mathrm{Re}(\alpha_{mj}\beta_{mk}^*A_{j\alpha}B_{k\alpha})$$
$$+\delta(\varepsilon-E_j)\delta(\varepsilon-E_k)\beta_{mj}\beta_{mk}^*B_{j\alpha}B_{k\alpha}^*] \tag{5.6}$$

where the repeated index α is not summed over.

For the Rindler case the first two terms in (5.6) do not contribute since it is impossible to satisfy the delta functions (as $E_j>0$ and $\varepsilon>0$) and so the transition probability is essentially given by $\Sigma_m|\beta_{mj}|^2$. Indeed this comment will apply to any static detector in a static space-time. This clearly supports the interpretation that the Rindler detector is responding purely to the Rindler particle content of the Minkowski vacuum. On the other hand, for the rotating case $\beta_{mj}=0$, so only the $|\alpha_{mj}|^2$ term in (5.6) contributes to the transition probability. To interpret this result we turn to the transition amplitude. From (5.4) we can see that the amplitude $A(F,\alpha)$ is composed of two terms: the first term, $\delta(\varepsilon+E_j)\langle F|\hat{b}_j^\dagger|G\rangle$ corresponds to radiation emitted by the detector and describes a transition in which the external field acquires a K particle. The second term, $\delta(\varepsilon-E_j)\langle F|\hat{b}_j|G\rangle$, corresponds to the absorption of a particle by the detector and describes a transition in which the global field loses a K particle. As noted above, for a static detector the first term vanishes. Moreover, when $|G\rangle=|K\rangle$ the second term clearly vanishes. In general, both terms are present and together lead to the standard quantum mechanical interference term in the probability (5.6).

Returning to the case of the rotating detector, only the first term in (5.4) will contribute: all excitations in the rotating detector are due entirely to the recoil from the radiation it emits. This interpretation is supported by the observation that the integral in (4.3) is identical to that obtained by Schwinger (1954) when he calculated the first-order quantum corrections to the synchrotron radiation emitted by an electron moving in a circle under the influence of a constant magnetic field, in the approximation that the electron is spinless. For the finite detector of § 4 we noted that transitions to states with negative j are suppressed relative to states with positive j. Thus, on average the radiation emitted by the detector will be such as to slow it down. Clearly it is necessary to supply energy from some external source to keep the detector rotating at a constant angular velocity and, indeed, it is this source which must supply the energy which is radiated by the detector. (We note in passing that this argument illustrates a general advantage of using a finite detector, namely, that one can consider the momentum transfer between the detector and the external field.)

For the Rindler detector only the second term in (5.4) will contribute. From an accelerated observer's point of view this term can be interpreted simply as corresponding to the absorption of a particle of energy ε from the thermal bath of particles present. On the other hand, in terms of Minkowski particles the final state is very complicated: the state $\hat{b}_j|M\rangle = \beta^*_{mj}\hat{a}^\dagger_m|M\rangle$ contains infinitely many Minkowski particles in a highly correlated superposition. Hence from an inertial observer's point of view the detector will emit a complicated flux of particles and not just a single Minkowski particle as stated by Unruh (1976).

The above observations show how, at a complete level, one is able to relate fluctuometer response to the particle content of a state, at least in the simple circumstances discussed in this section: after a particle detection the number of particles in the external field decreases so that on average the probability of a subsequent detection falls. After a radiation recoil excitation the number of particles in the external field increases so that on average the probability of a subsequent detection rises. Thus, by using several detectors (as one realistically would) and calculating the correlations between them, one can determine which excitations are spurious radiation effects and which correspond to particle detections. We stress that in considering the response of a classical detector one must also, in general, consider the effects of classical radiation. The only difference in the quantum case is that there are additional radiation effects which are present even when no particles are present. These additional effects can be related to the purely quantum phenomenon of radiation by moving mirrors (Unruh and Wald 1982).

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Appendix

In this appendix we discuss properties of the detector modes ψ_{kmq} used in the rotating detector calculations of § 4. We recall that

$$\psi_{kmq} = N_{kmq} \exp(-\mathrm{i}(\omega - m\Omega)t) \exp(\mathrm{i}m\theta) \sin(k[z+h]) C_{|m|}(qr) \tag{A1}$$

where $m = \pi j/\alpha, j \in \mathbb{Z}; k = \pi n/2h, n = 0, 1, 2, \ldots; \omega = (q^2+k^2)^{1/2}$ and N_{kmq} is a normalisation factor. $C_{|m|}(qr)$ is a cylinder function of order $|m|$ such that

$$C_{|m|}(qa) = C_{|m|}(qb) = 0. \tag{A2}$$

Firstly, we note that the zeros of cylinder functions of order r are interlaced with the zeros of J_r (Erdelyi *et al* 1953). Hence, if $j_{r,n}$ denotes the nth positive zero of J_r then since qb must be at least the second positive zero of the function $C_{|m|}(x)$ it follows that $qb > j_{|m|,1}$. Moreover, it is known that $j_{r,1} > r$ (Erdelyi *et al* 1953), and so $qb > |m|$.

Thus, if $\Omega b < 1$ (the condition that the outer edge of the box should travel at less than the speed of light) then $q > |m|\Omega$ and so $\varepsilon = (q^2 + k^2)^{1/2} - m\Omega > 0$.

Secondly, we note that for large n,

$$j_{r,n} = (n + \tfrac{1}{2}r - \tfrac{1}{4})\pi + \mathrm{O}(1/n),$$

and for large r,

$$j_{r,n} = r + \mathrm{O}(r^{1/3}).$$

Hence, it is a good approximation to assume that the positive zeros of a cylinder function $C_r(x)$ are uniformly distributed over the region $x > r$. Determining the density of states under this approximation is straightforward. The result is given in equation (4.4).

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